Video Two: Algebraic and Transcendental Numbers.

Another, more subtle partition.

Hidden deep within a subset of functions is a very special partition. It was discovered in the late 17th century and has been fascinating mathematicians ever since.

We'll look first at Polynomial functions. Which is to say that we will be selecting functions for this proper subset by looking at their FORM. If a function has the right polynomial form, then it's in.

The right polynomial form is an added, finite collection of monomials with rational coefficients. So, what is a monomial? " ax^n " where the coefficient is a real number and the exponent is a whole number.

$3x^2$ is a monomial and so is 12 ($12x^0$). So $f(x)=3x^2+12$ is the right form for our set.

Here are some functions that do not have the right polynomial form:

$f(x)=5^x$

$f(x)=2x^{0.5}$

$f(x)=|x+3|$

$f(x)=\dfrac{5x}{x+2}$

$f(x)=\pi x+\sqrt{5}$

Here are some functions that DO have the right polynomial form:

$f(x)=5$

$f(x)=-.25x+12$

$f(x)=x^2+1$

$f(x)=-4x^{101}+5x^{37}-6$

These are all polynomials with rational coefficients, too. More restrictive than just polynomials.

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$f(x) = 12x^9 - 11x + 7$

Now some of these have *x*-axis intercepts, otherwise known as "roots".

Let's take a minute and look at some polynomial functions with roots.

Note that the horizontal line $y = 5$, is a polynomial with rational coefficients, but it does not contribute to the set of roots that we are going to learn about.

I want a collection of numbers that are the roots to polynomials with rational coefficients.

So let's start exploring that set.

Suppose I take a very familiar type of polynomial: a line with $0 < |m| < \infty$. We've spent some time with these. And their *x*-axis intercepts are just the kind of roots we want to focus on.

$$f(x) = 0.25x + \pi$$

Is this a polynomial? With rational coefficients?

$$f(x) = 0.25x + 0.1$$

How do you solve for the x-axis intercepts? (*i.e*. solve for the root?)

Well, at an *x*-axis intercept the *y* value is zero!

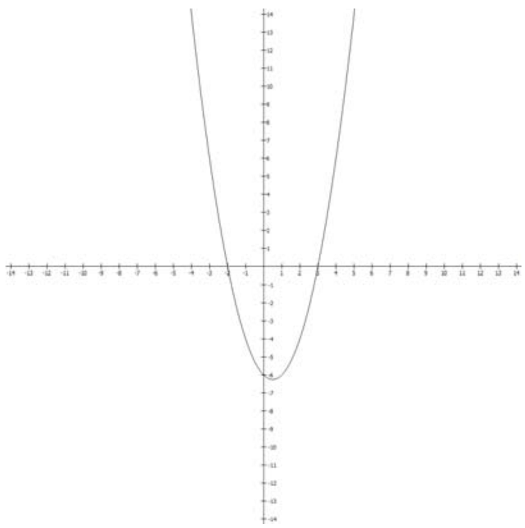
$$0 = 0.25x + 0.1$$

This number is called an ALGEBRAIC NUMBER. It is the root of a polynomial with rational coefficents (and recall, whole number exponents comes with the definition of "polynomial")

This is not very startling. ALL rational numbers are the roots of some polynomial with rational coefficients.

Here’s some more not-surprising ones:

$f(x) = (x+2)(x-3) = x^2 - x - 6$



Let’s look at where the roots are, and the *y*-intercept…the actual numbers and where they are on the graph.

Which are the ones we are looking at?

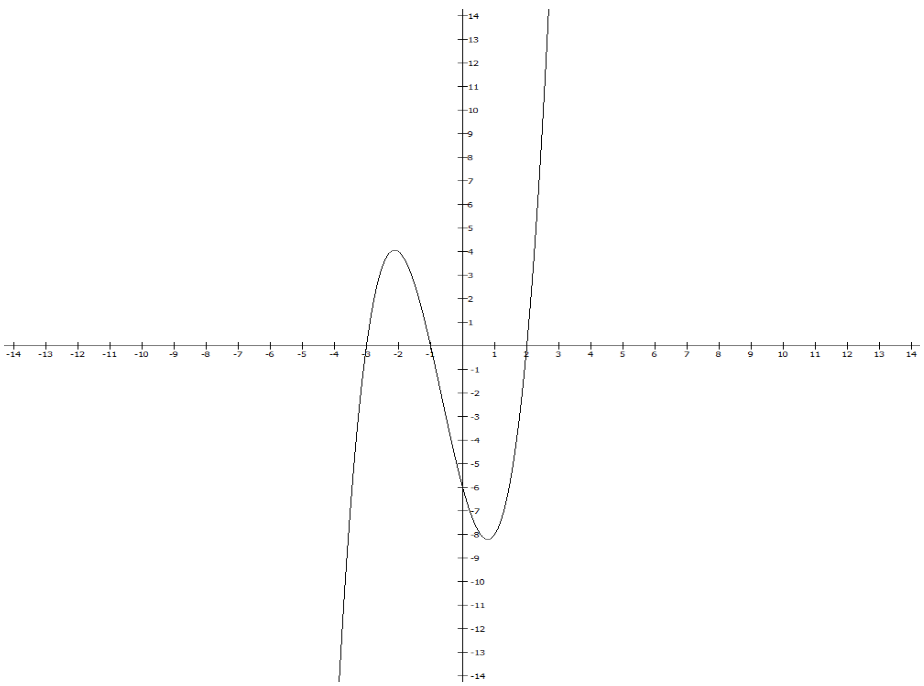
These are algebraic numbers.

Here's another one

$$f(x) = (x+1)(x-2)(x+3) = x^3 + junk - 6$$

What are the roots? We can tell from the factored form…yay for factoring!

Now let's look at the graph:



Is this a one-to-one function?

What are the algebraic numbers here?

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Now for some surprises!

Let's look at:

$$f(x) = x^2 - 2$$

$$0 = x^2 - 2$$

Is this a polynomial with rational coefficients?

What are the roots?

Oh! What kind of numbers are those?

And they ARE algebraic numbers.

So let's list the algebraic numbers so far:

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Another surprise is:

$$f(x) = x^2 + 1$$

$$0 = x^2 + 1$$

Right kind of polynomial? What are the algebraic numbers?

What are we looking at here for a subset?

Now this is a proper subset of the Complex Numbers. What are the numbers OUTSIDE this set of algebraic numbers called? TRANSCENDENTAL numbers

Let's draw a set diagram:

Do you see the partition?

What are some of the transcendental numbers and when were they identified?

The first glimmer of algebraic and transcendental numbers came from Leibniz in 1682. Lambert showed in 1761 pi id transcendental. The subject is intrinsically difficult. Hermite showed e is transcendental in 1873. The next big breakthrough was in about 1845 with Liouville came up with his own transcendental numbers. Cantor proved that there are more transcendental numbers than natural numbers in 1878.

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So here we have a subtle partition, and a fairly recent one! And it crosses boundaries that people think of as fixed and immutable. It really makes you think harder about what it means when you say two numbers are similar!